Solution to
Fourth International Mathematics Assessment for Schools
Round 1 of Upper Division

1. What is the value of 2015 + 1520 + 5201?

（A）8236　（B）8506　（C）8736
（D）8836　（E）9716

【Suggested Solution 1】
2015 + 1520 + 5201 = 8736. Hence, we select (C).

【Suggested Solution 2】
2015 + 1520 + 5201 = 2015 + 0152 + 1520 + 5201－152 = 8888－152 = 8736.
So, we select (C).

Answer: (C)

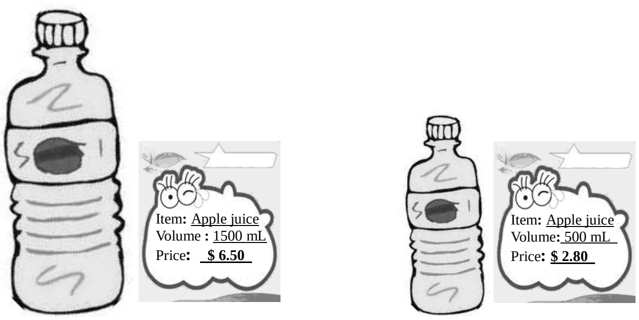
2. The three numbers in each of the following groups are multiplied together. Which group yields 2014 as the product?

（A）6、17、59　（B）4、17、53　（C）2、13、59
（D）2、19、53　（E）2、23、29

【Suggested Solution】
The units digit in the product of three numbers in option (A) is 8, it is unreasonable!
The product of three numbers $4 \times 17 \times 53 > 60 \times 50 = 3000$, so option (B) did not meet the condition!
The product of three numbers $2 \times 13 \times 59 < 30 \times 60 = 1800$, so option (C) is also unreasonable!
While the product of those three numbers $2 \times 23 \times 29 < 50 \times 30 = 1500$, so option (E) is not our selection!
Only the product of three numbers $2 \times 19 \times 53 = 2014$ in option (D) is the same as the requirement of the problem. Hence, we select (D).

Answer: (D)

3. A large bottle of apple juice costs 6.5 dollars while a small bottle of apple juice costs 2.8 dollars. How many dollars less is the cost of a large bottle compared to the total cost of three small bottles?



（A）1.9　（B）2.1　（C）2.3　（D）2.8　（E）3.7

【Suggested Solution】

The cost of 3 small bottles of apple juice is \$2.8 × 3 = \$8.4. Hence buying 1 large bottle of apple juice is less than buying 3 small bottles of apple juice by \$8.4－\$6.5 = \$1.9. Therefore, we select option (A).

Answer: (A)

4. Which of the following differences has the smallest value?

(A) $1-\frac{1}{2}$ (B) $\frac{1}{2}-\frac{1}{3}$ (C) $\frac{1}{3}-\frac{1}{4}$

(D) $\frac{1}{4}-\frac{1}{5}$ (E) $\frac{1}{5}-\frac{1}{6}$

【Suggested Solution】

Since $1-\frac{1}{2}=\frac{1}{2}>\frac{1}{2}-\frac{1}{3}=\frac{1}{6}>\frac{1}{3}-\frac{1}{4}=\frac{1}{12}>\frac{1}{4}-\frac{1}{5}=\frac{1}{20}>\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$. Hence, we select (E).

Answer: (E)

5. The two stars in the diagram represent the same number. The sum of the three numbers in the second row is equal to twice the sum of the three numbers in the first row. What number does each star represent?

5	6	☆		
		☆	19	20

(A) 7 (B) 8 (C) 13 (D) 17 (E) 18

【Suggested Solution】

From the given information, it shows the sum of the three numbers in the second row is equal to twice the sum of the three numbers in the first row, it follows that the difference of the sum of three numbers in the second row and the sum of three numbers in the first row equal the sum of three numbers in the first row, that is; the difference of these two rows is 19 + 20－5－6 = 28, then we have ☆ = 28－5－6 = 17. Thus, we select option (D).

Answer: (D)

6. A sack of flour costs 800 dollars and a sack of rice costs 500 dollars. Anne buys several sacks of each kind and spends 3400 dollars. How many sacks of flour does she buy?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

【Suggested Solution】

Since 800 × 5 = 4000 > 3400, so Anne cannot buy more than 5 sacks of flour. \$3400 deducted to the total amount she spend on flour must be multiple of \$500, then Ann can at most buy 3 sacks of flour and 2 sacks of rice; that is, 800 × 3 + 500 × 2 = \$3400. Hence, we select (C).

Answer: (C)

7. When two numbers are divided by 5, the respective remainders are 4 and 2. What is the remainder when the sum of the two numbers is divided by 5?

（A）0 （B）1 （C）2 （D）3 （E）4

【Suggested Solution】

The sum of the remainders of two numbers is 2 + 4 = 6, continue dividing this remainder by 5, we have the final remainder 1. Hence, we select option (B).

Answer: (B)

8. The numbers 38, 79, 17, 43, 74, 96 and 87 are rearranged so that starting from the second one, its tens digit is equal to the units digit of the preceding number. Which number is in the fourth place after the rearrangement?

（A）38 （B）43 （C）17 （D）96 （E）87

【Suggested Solution】

After rearranging the 7 numbers, except the tens' digit of the first number and the units digit of the last number, all the other digits must appear even number of times. After going over all the 7 numbers, we discover that digit 1 and digit 6 appear once. So that 17 must be the first number and the arrangement must be: 17, 74, 43, 38, 87, 79 and 96. Thus, the number in the fourth place must be 38. Hence, we select option (A).

Answer: (A)

9. To visit a friend, Rod must take the bus to the nearest Metro station, and this takes 15 minutes. He has to ride the Metro train for 20 stops, each taking 2.5 minutes. He also has to change trains twice, and it takes 3 minutes each time. Finally, after exiting the Metro, he still has to walk another 12 minutes before reaching his friends place. How many minutes does Rod have to spend traveling to his friend's house?

（A）55 （B）67 （C）80 （D）83 （E）90

【Suggested Solution】

From the given information, Rod must spend a total of $15 + 2.5 \times 20 + 3 \times 2 + 12 =$ 83 minutes. Hence, we select option (D) .

Answer: (D)

10. A table has 3 rows and 3 columns. In each square, we put in the product of the row number and the column number of that square. For instance, the number in the third row and the second column is $3 \times 2 = 6$, as shown in the diagram. These nine numbers are then arranged in order of size, starting with the smallest. What is the fifth number in the arrangement?

	6	

（A）2 （B）3 （C）4 （D）5 （E）6

【Suggested Solution】

From the given information, each number inside the small squares must be filled in as follows,

1	2	3
2	4	6
3	6	9

these numbers are arranged from the smallest to the largest as in the following:

1, 2, 2, 3, 3, 4, 6, 6, 9.

Therefore, the fifth number in the arrangement is 3. Hence, we select (B).

Answer: (B)

11. Oliver arranges his toy ducks and toy turtles in a row as shown in the diagram. He wishes to have all the toy ducks on the left and all the toy turtles on the right. He may switch the position of any two adjacent toys. What is the minimum number of switches he will require?

(A) 15 (B) 16 (C) 17 (D) 18 (E) 19

【Suggested Solution】

We know that for each switch of two adjacent toys may make one toy duck move at most one position to the left, let us name the position of each toy from left to right as 1 to 11. Our main target is to arrange the toys in the manner that all 6 toy ducks will be on the left side. Since the initial position of 6 toy ducks are 1, 3, 5, 8, 10, 11, then we need to operate at least $(1-1)+(3-2)+(5-3)+(8-4)+(10-5)+(11-6)=17$ times of switches in order to reach our goal. Hence, we must perform switches at least 17 times. So, we select (C).

Answer: (C)

12. The sum of one-fifth of a non-negative integer and one-third of another non-negative integer is 1. What is the maximum value of the sum of these two non-negative integers?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

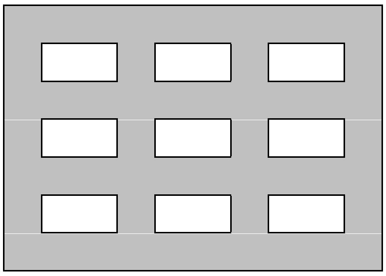
【Suggested Solution】

Assume the two non-negative integers represent as a and b, then $\frac{a}{5}+\frac{b}{3}=1$. So the possible values of b can only be 0, 1, 2, 3.

If $b=0$, then $a=5$. If $b=1$, then $a=\frac{10}{3}$ which is not an integer. If $b=2$, then $a=\frac{5}{3}$ not an integer again. If $b=3$, then $a=0$. Thus, there are only two possible values of $a+b$, they are $5+0=5$, $0+3=3$. This implies the maximum sum of $a+b$ is 5. Hence, we select (A).

Answer: (A)

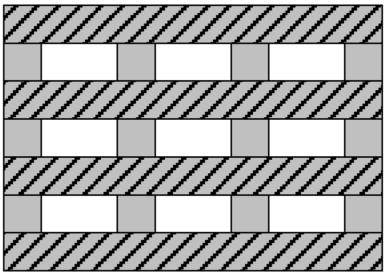
13. Max has a gray paper strip of width 5 cm. He cuts out some pieces and tapes them on the wall, to form a 50 cm by 35 cm window frame, as shown in the diagram, where each white area is 5 cm by 10 cm. What is the minimum length, in cm, of the paper strip required?



（A）260　　（B）280　　（C）300　　（D）340　　（E）360

【Suggested Solution】

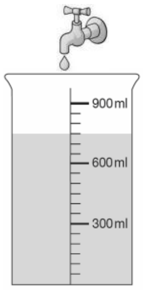
Refer to the diagram below. The paper strip in the window frame was divided into 16 pieces with 4 pieces of length 50 cm and 12 pieces of length 5 cm.



Thus, Max must cut a minimum of $50 \times 4 + 5 \times 12 = 260$ cm paper strips. Hence, we select option (A).

Answer: (A)

14. The tap is leaking at the rate of one drop per second. The volume of each drop is 0.05 mL. At 9 pm, Wendy puts an empty measuring cup under the tap. Some time during the night, she finds the cup partially filled, as shown in the diagram. No water is lost from the cup. At what time is the water level in the measuring cup closest to the time in the following?



（A）23：10　　（B）00：30　　（C）01：10

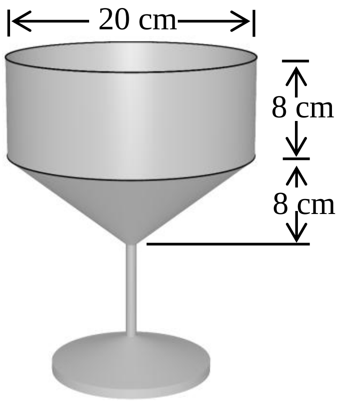
（D）01：50　　（E）02：10

【Suggested Solution】

Each scale of the measuring cup in the diagram is $300 \div 6 = 50$ mL, this implies that the water in measuring cup is 750 mL. From the given information, we know the tap has leaked 750 mL in around $750 \div 0.05 = 15000$ seconds = 4 hours 10 minutes, which is nearer to 01：10. Hence, select option (C).

Answer: (C)

15. The upper part of a glass is a cylinder of height 8 cm and diameter 20 cm. The middle part is a cone of height 8 cm and diameter 20 cm. The bottom part is a solid stem. Correct to one decimal place, what is the capacity, in cm^3, of the cup? Take π=3.14.



(A) 837.3 cm^3 (B) 1674.7 cm^3
(C) 2512.0 cm^3 (D) 3349.3 cm^3
(E) 5024.0 cm^3

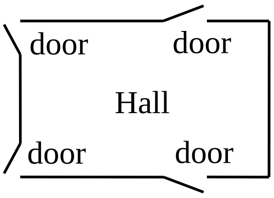
【Suggested Solution】

The volume of the cylinder is $3.14\times10^2\times8=2512\,\text{cm}^3$, the volume of the cone is $\frac{1}{3}\times3.14\times10^2\times8\approx837.3\,\text{cm}^3$. Therefore, the volume of the cup is 2512 + 837.3 = 3349.3 cm^3. Hence, we select (D).

Answer: (D)

16. A hall has four doors. Lea may enter the hall using any of them, and exit the hall using any of them. In how many different ways can she enter and exit the hall?

(A) 4 (B) 8 (C) 12 (D) 16 (E) 24

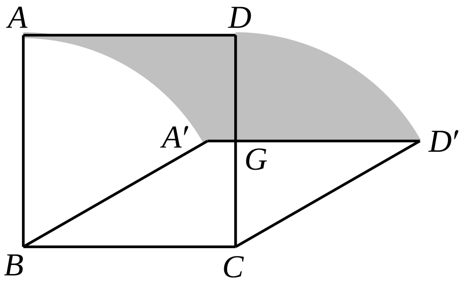


【Suggested Solution】

Lea enters the hall from one of the 4 doors and exits using any of the 4 doors, thus there are 4 different ways of enter the hall and also 4 different ways to exit the hall. Hence, Lea has a total of $4\times4=16$ different ways enter and/or exit the hall. Hence, we select (D).

Answer: (D)

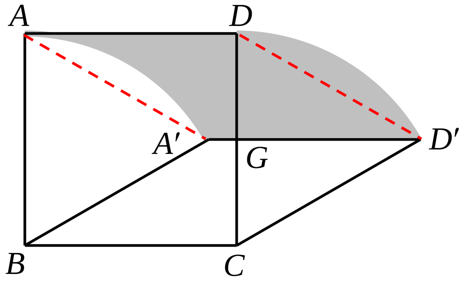
17. *ABCD* is a square of side length 10 cm. The segment *BC* is fixed. The segment *AD* moves in the plane to the segment *A′D′* so that the lengths *AB*, *DC* and *AD* do not change. What is the area, in cm^2, of the shaded region in the diagram when the segment *A′D′* intersects the segment *CD* at its midpoint *G*?



(A) 50 (B) $\frac{50\pi}{3}$ (C) 60 (D) 100 (E) $\frac{100\pi}{3}$

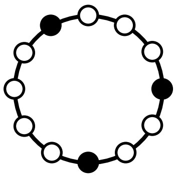
【Suggested Solution】

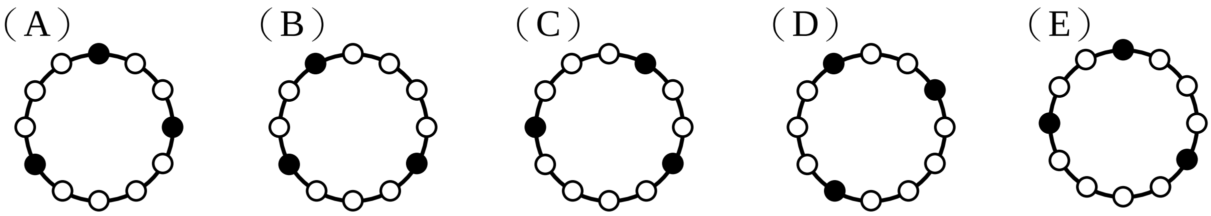
Connect AA' and $D'D$, Apply Cut and Paste method we know the area of the shaded region equals to the area of parallelogram $AA'D'D$, in parallelogram $AA'D'D$ we have base $AD = 10$ cm, altitude $GD = 5$ cm and so, the area is $10 \times 5 = 50$ cm^2. Hence, we select option (A).



Answer: (A)

18. On a table there is a ring, there are 12 equally spaced beads on the ring, 3 of which are black, as shown in the diagram. Which of the following five figures cannot be obtained from the given figure by rotating the ring on the table?



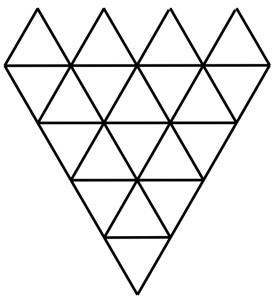


【Suggested Solution】

Observe the arrangement as regards the number of small white circles between two black circles of the given diagram in counterclockwise direction: four white small circles, two small white circles and then three small white circles. In all the five options, only option (E) does not comply with this kind of arrangement. Hence, we select (E).

Answer: (E)

19. The figure in the diagram is formed from 20 equilateral triangles of equal sizes. How many equilateral triangles of any size does it contain? The triangles may overlap.



（A）20　（B）26　（C）30　（D）33　（E）39

【Suggested Solution】

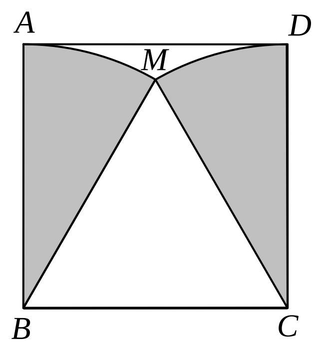
Let the side length of the smallest equilateral triangle as 1 unit. We discover there are 20 equilateral triangles whose side length is 1 unit, there are 9 equilateral triangles whose side length is 2 units, 3 equilateral triangles whose side length is 3 units and 1 equilateral triangle whose side length is 4 units. Therefore, there are 20 + 9 + 3 + 1 = 33 different sizes of equilateral triangles. Hence, we select option (D).

Answer: (D)

20. The sectors *MAB* and *MCD* are inside the square *ABCD* of side length 10 cm, as shown in the diagram. What is the total area, in cm^2, of these two sectors, correct to 1 decimal place? Take π=3.14.

（A）52.3 （B）78.5 （C）104.7

（D）157.0 （E）314.0



【Suggested Solution】

From the given information, $\triangle BCM$ is an equilateral triangle, then $\angle ABM = \angle MCD = 30°$. Total area of the two sectors is

$3.14 \times 10^2 \times \frac{30}{360} \times 2 \approx 52.3\,cm^2$. Hence, option (A).

Answer: (A)

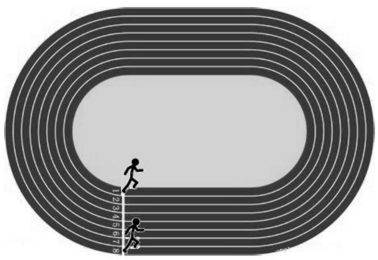
21. In the sequence 1, 1, 2, 3, 5, 8, 13, 21, ⋯, each term starting from the third one is the sum of the preceding two. How many among the first 2014 terms are divisible by 4?

【Suggested Solution】

The remainder when each term in the sequence divided by 4 is listed as 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0, ⋯. We discover the recurring period is 6 and one of the remainder is 0, it means one term in every recurring period in the sequence is divisible by 4.
Since 2014 = 335 × 6 + 4, it means that in the first 2014 terms in the sequence, there are 335 complete periods with an incomplete period of four remaining terms.
Therefore, there are 335 terms in the sequence that are divisible by 4.

Answer: 335

22. The inside lane of a track has length 400 m and the outside lane has length less than 500 m. From the marked line, as shown in the diagram, Max and Lynn start running counterclockwise along the track at the same time. Max runs at constant speed on the inside lane. Lynn, whose constant speed is 3 times that of Max, runs on the outside lane. The first time both are back together at the marked line, Max has completed 3 laps. What is the length, to the nearest m, of the outside lane?



【Suggested Solution】

From the given information, when Max and Lynn return back at the original marked line, Max has run 400 × 3 = 1200 m while Lynn has run 1200 × 3 = 3600 m. Likewise, Lynn has finished exactly integer number of laps.

When Lynn ran 9 laps or more laps, then the length of the outside lane would be less than or equal to 3600 ÷ 9 = 400 m, which didn't meet the condition of the problem.

When Lynn ran exactly 8 laps, then the length of outside lane was 3600 ÷ 8 = 450 m.

When Lynn ran 7 laps or less laps, then the length of outside lane was equal or more than $\frac{3600}{7} > 500$ m. Thus, a contradiction to the given information.

Therefore, the length of outside line is 450 m.

Answer: 450

23. How many two-digit prime numbers are there if the three-digit number obtained by inserting a 1 between the two digits is also a prime number?

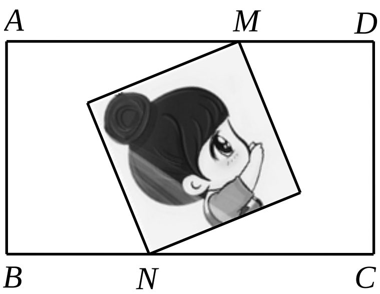
【Suggested Solution】

First let us search for all the two-digit prime numbers that divided by 3 that will give a remainder of 1 or 2. If $\overline{ab}$ divided by 3 gives a remainder of 2, then the three-digit number $\overline{a1b}$ is divisible by 3, so $\overline{a1b}$ cannot be a prime number. Hence, the two-digit prime number $\overline{ab}$ when divided by 3 will yield a remainder of 1. Consider the following two-digit prime number: 13, 19, 31, 37, 43, 61, 67, 73, 79, 97 and insert the digit 1 to do the verification one at a time all the three-digit number in the form of $\overline{a1b}$ if it is a prime number or not: 113 is a prime number, 119 = 7 × 17, 311 is a prime number, 317 is a prime number, 413 = 7 × 59, 611 = 13 × 47, 617 is also prime number, 713 = 23 × 31, 719 is a prime number while 917 = 7 × 131.

Therefore, there are 5 different two-digit prime numbers $\overline{ab}$.

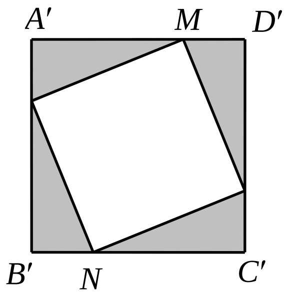
Answer: 005

24. An 8 cm by 8 cm photograph is loose inside an 18 cm by 10 cm frame *ABCD*. However, the point *M* remains on *AD* and the point *N* remains on *BC*, as shown in the diagram. What is the area, in cm^2, of the region inside the frame which is never covered by the photograph?



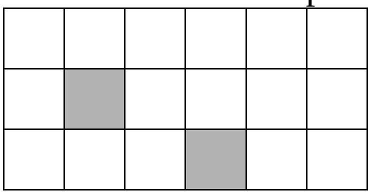
【Suggested Solution】

From the given problem, the region inside the frame that has never been covered by the photograph is equivalent as the shaded parts in the diagram at the right. By Property of Symmetry for a square, we know that *A′B′C′D′* is also a square, and the side length is 10 cm. Then the area of the shaded region is $10^2 - 8^2 = 36\,cm^2$. Hence, the area of region inside the frame which is never covered by the photograph is 36 cm^2.



Answer: 036

25. How many pairs of unit squares in a 3 by 6 table are such that they have no common points? The diagram shows one such pair.

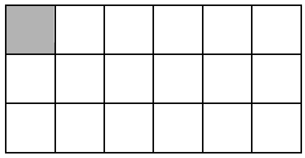


【Suggested Solution 1】

When the two selected unit squares are on the same column, then one must be in the first row and the other in the third row. Therefore, there are 6 possible pairs.

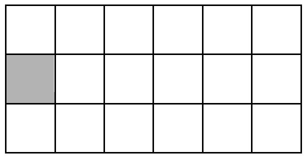
When two unit squares selected are not in the same column, assume that one of the unit square is to the left of the other and below are the possible cases:

(1) When the left unit square is located on the row 1.



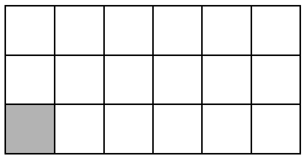
When the left unit square is on the 1st column, then other unit square can be located on the 2nd column of row 3 or on any unit square on the 3rd , 4th, 5th and 6th column. There are a total of 13 possible pairs. Likewise, when the unit squares located on any unit squares of other column. There are a total of 10 + 7 + 4 + 1 = 22 possible pairs.

(2) The left unit square is located on row 2.



When the left unit square is on the 1st column, then the other unit square can be located on any other location of the 3rd, 4th, 5th and 6th column. There are a total of 12 possible pairs. Likewise, the unit square is located on any other column, there are a total of 9 + 6 + 3 = 18 possible pairs.

(3) The left unit square is located on the 3rd row.



The situation is same as that of Case 1, we have a total of 13 + 22 = 35 possible pairs of unit squares.

In summary, we have a total of 6 + 35 + 30 + 35 = 106 distinct pairs of unit square.

【Suggested Solution 2】

Select any a pair of unit square in a 3 × 6 table, we have a total of 18 × 17 ÷ 2 = 153 possible pairs. Hence, to select two unit squares that don't have a common point, so we must deduct those two unit squares having a common edge or common vertex. There are 5 × 3 + 2 × 6 = 27 pair of unit squares having a common edge while there are 2 × (1 + 2 + 2 + 2 + 2 + 1) = 20 pair of unit squares having a common vertex. Therefore, there are a total of 153 − 27 − 20 = 106 possible pair of unit squares.

Answer: 106

* * *